



*Qweak collaboration meeting  
William & Mary, Nov. 15, 2012*

# $\gamma Z$ interference corrections to PVES: an update

*Wally Melnitchouk*

*Nathan Hall (Adelaide), Peter Blunden (Manitoba),  
Tony Thomas (Adelaide), Ross Young (Adelaide)*

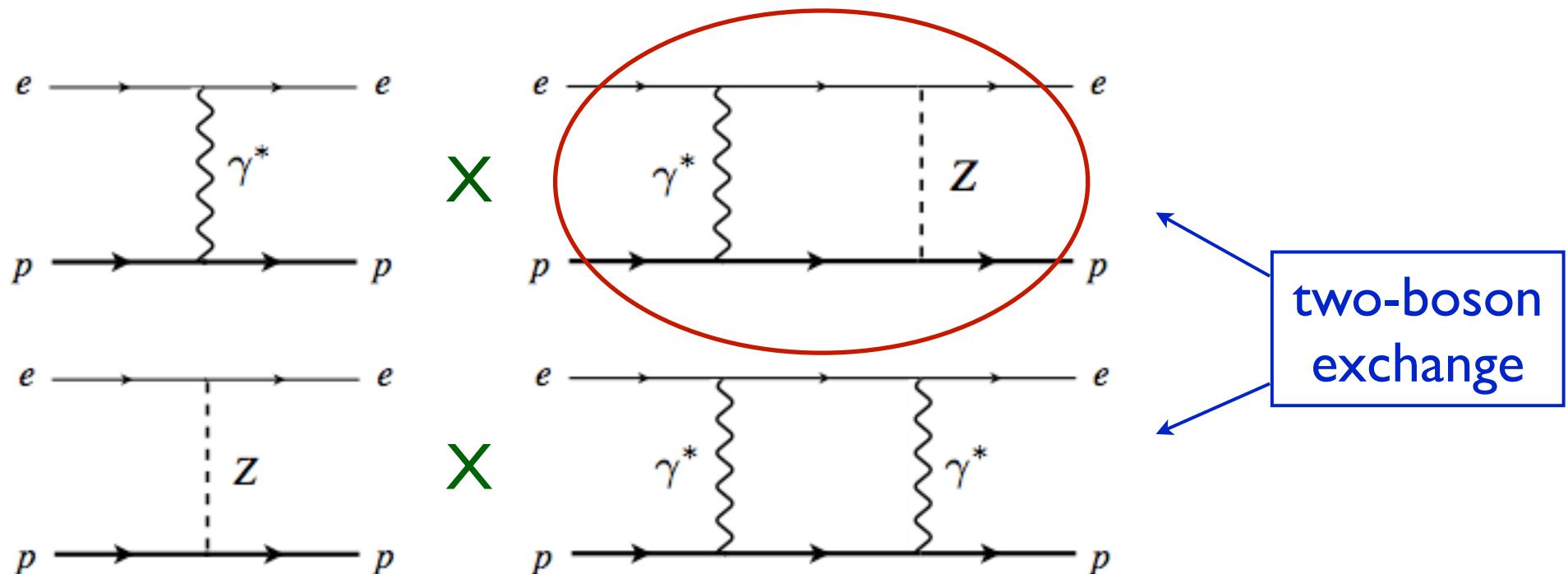
*“AJM” collaboration*

# Corrections to proton weak charge

## ■ Including higher order radiative corrections

$$\begin{aligned} Q_W^p &= (1 + \Delta\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta'_e) \\ &\quad + \square_{WW} + \square_{ZZ} + \square_{\gamma Z} \xleftarrow{\text{box diagrams}} \\ &= 0.0713 \pm 0.0008 \end{aligned}$$

Erler et al., PRD 72, 073003 (2005)



# Corrections to proton weak charge

## ■ Including higher order radiative corrections

$$\begin{aligned} Q_W^p &= (1 + \Delta\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta'_e) \\ &\quad + \square_{WW} + \square_{ZZ} + \square_{\gamma Z} \xleftarrow{\text{box diagrams}} \\ &= 0.0713 \pm 0.0008 \end{aligned}$$

*Erler et al., PRD 72, 073003 (2005)*

→ **WW and ZZ box diagrams dominated by short distances, evaluated perturbatively (WW box gives  $\sim 25\%$  correction!)**

→  **$\gamma Z$  box diagram sensitive to long distance physics, has two contributions**

$$\square_{\gamma Z} = \square_{\gamma Z}^A + \square_{\gamma Z}^V$$

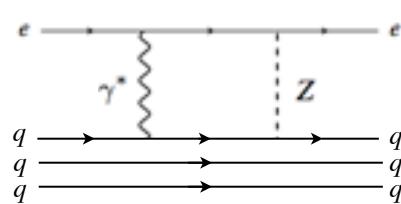
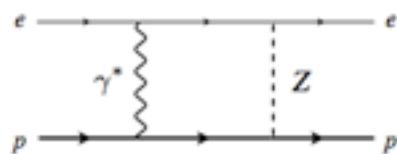
↑  
vector  $e$  – axial  $h$   
(finite at  $E=0$ )

↑  
axial  $e$  – vector  $h$   
(vanishes at  $E=0$ )

# Axial $h$ correction

- Axial  $h$  correction  $\square_{\gamma Z}^A$  dominant  $\gamma Z$  correction in atomic parity violation at very low (zero) energy

→ computed by Marciano & Sirlin (1980s) as sum of two parts:



- ★ low-energy part approximated by *Born contribution* (elastic intermediate state)
- ★ high-energy part (above scale  $\Lambda \sim 1$  GeV) computed in terms of scattering from *free quarks*

$$\begin{aligned}\square_{\gamma Z}^A &= \frac{5\alpha}{2\pi} (1 - 4 \sin^2 \theta_W) \left[ \ln \frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda) \right] \\ &\approx 0.0052(5)\end{aligned}$$

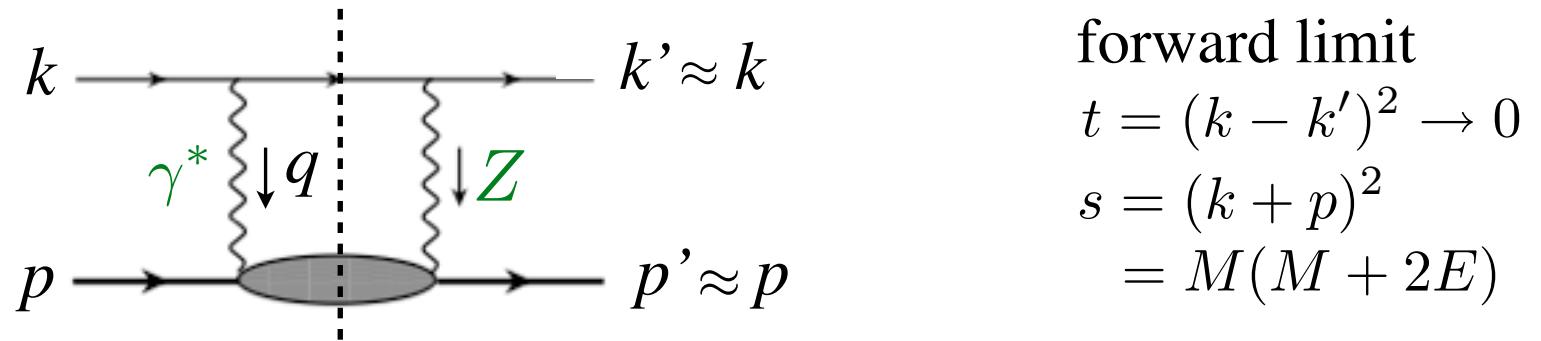
short-distance      long-distance  $\approx 3/2 \pm 1$

Marciano, Sirlin, PRD 29, 75 (1984); Erler et al., PRD 68, 016006 (2003)

# Axial $h$ correction

- Axial  $h$  correction  $\square_{\gamma Z}^A$  dominant  $\gamma Z$  correction in atomic parity violation at very low (zero) energy

→ evaluate using *forward dispersion relations* with realistic input (inclusive structure function)



- ★ axial  $h$  contribution *antisymmetric* under  $E' \leftrightarrow -E'$ :

$$\Re e \square_{\gamma Z}^A(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2} \Im m \square_{\gamma Z}^A(E')$$

- ★ negative energy part corresponds to crossed box (crossing symmetry  $s \rightarrow u$ )

# Axial $h$ correction

- Imaginary part given by interference  $F_3^{\gamma Z}$  structure function

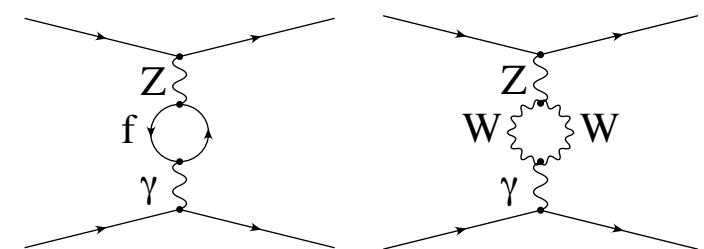
$$\begin{aligned} \text{Im } \square_{\gamma Z}^A(E) &= \frac{1}{(2ME)^2} \int_{M^2}^s dW^2 \int_0^{Q_{\max}^2} dQ^2 \frac{v_e(Q^2) \alpha(Q^2)}{1 + Q^2/M_Z^2} \\ &\quad \times \left( \frac{2ME}{W^2 - M^2 + Q^2} - \frac{1}{2} \right) F_3^{\gamma Z} \end{aligned}$$

with  $v_e(Q^2) = 1 - 4\kappa(Q^2) \sin^2 \theta_W(Q^2)$

- scale dependence of  $v_e, \alpha$  given by vacuum polarization corrections, e.g.

$$\frac{\alpha}{\alpha(Q^2)} = 1 - \Delta\alpha_{\text{lep}}(Q^2) - \Delta\alpha_{\text{had}}^{(5)}(Q^2)$$

$$\alpha^{-1}(M_Z^2) = 128.94$$



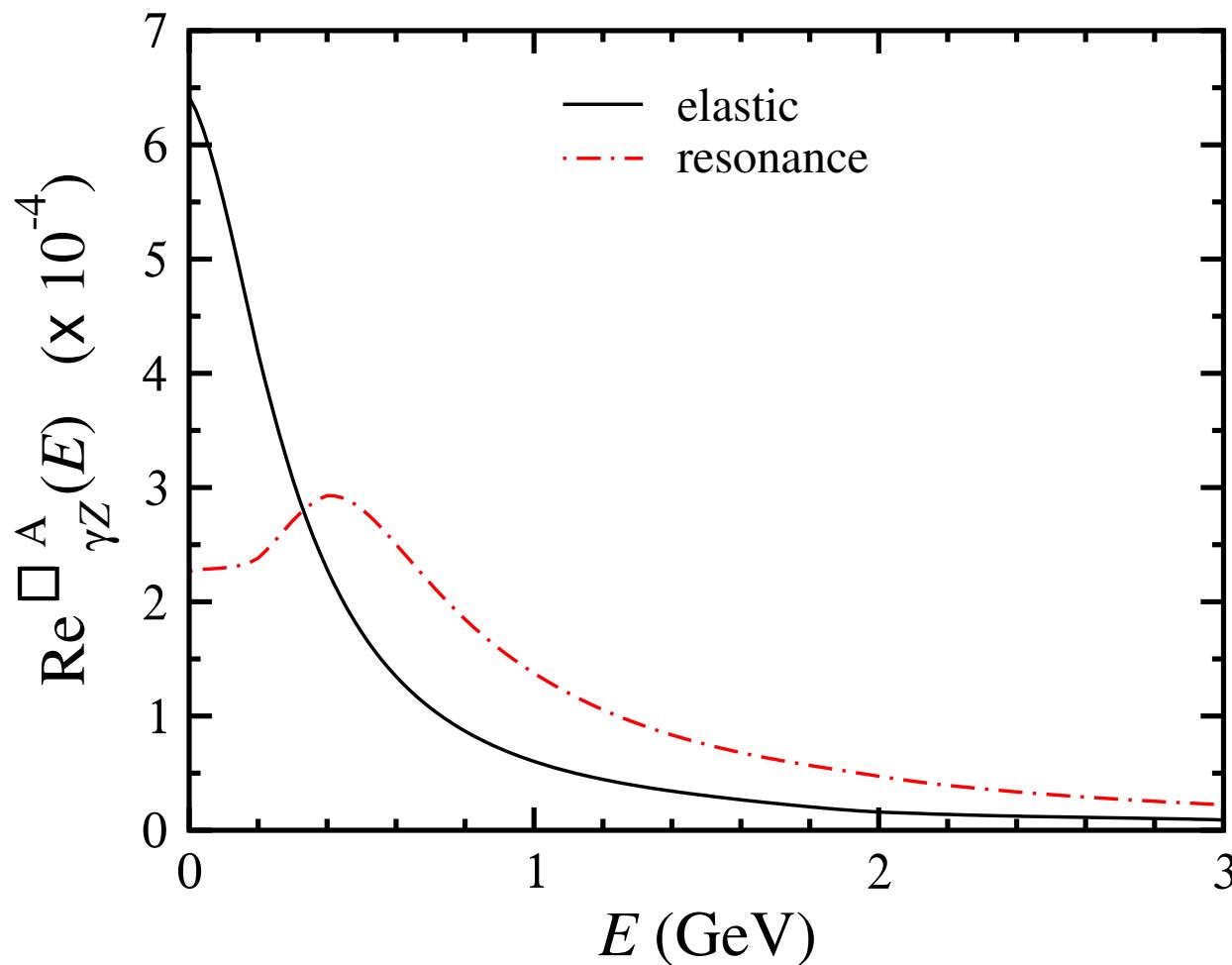
Jegerlehner, arXiv:1107.4683 [hep-ph]

... similarly for weak charges

# Axial $h$ correction

- ★ elastic part  $F_3^{\gamma Z(\text{el})} = -Q^2 G_M^p(Q^2) G_A^Z(Q^2) \delta(W^2 - M^2)$
- ★ resonance part from parametrization of  $\nu$  scattering data

Lalakulich, Paschos  
PRD 74, 014009 (2006)



Blunden, WM, Thomas  
PRL 107, 081801 (2011)

# Axial $h$ correction

- ★ DIS part dominated by leading twist PDFs at high  $W$  (small  $x$ )

e.g. at LO,  $F_3^{\gamma Z(\text{DIS})} = \sum_q 2e_q g_A^q (q(x, Q^2) - \bar{q}(x, Q^2))$

→ expand integrand in  $1/Q^2$  in DIS region ( $Q^2 \gtrsim 1 \text{ GeV}^2$ )

$$\begin{aligned} \mathcal{R}e \square_{\gamma Z}^{A(\text{DIS})}(E) &= \frac{3}{2\pi} \int_{Q_0^2}^{\infty} dQ^2 \frac{v_e(Q^2) \alpha(Q^2)}{1 + Q^2/M_Z^2} \\ &\times \left[ M_3^{\gamma Z(1)} - \frac{2M^2}{9Q^4} (5E^2 - 3Q^2) M_3^{\gamma Z(3)} \right] \end{aligned}$$

moments  $M_3^{\gamma Z(n)}(Q^2) = \int_0^1 dx x^{n-1} F_3^{\gamma Z}(x, Q^2)$

# Axial $h$ correction

## ■ Structure function moments

$$\underline{n=1} \quad M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3} \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right)$$

→  $\gamma Z$  analog of Gross-Llewellyn Smith sum rule

$$\Re e \square_{\gamma Z}^{A(\text{DIS})} \approx (1 - 4\hat{s}^2) \frac{5\alpha}{2\pi} \int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^2(1 + Q^2/M_Z^2)} \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right)$$



→ precisely result from Marciano & Sirlin!  
(works because result depends on lowest moment of  
*valence* PDF, with model-independent normalization!)

$$\underline{n=3} \quad M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3} (2\langle x^2 \rangle_u + \langle x^2 \rangle_d) \left( 1 + \frac{5\alpha_s(Q^2)}{12\pi} \right)$$

→ related to  $x^2$  -weighted moment of valence PDFs

## Axial $h$ correction

- ★ “DIS” region at  $Q^2 < 1 \text{ GeV}^2$  does not afford PDF description
  - in absence of data, consider models with general constraints
- ★  $F_3^{\gamma Z}(x_{\max}, Q^2)$  should not diverge in limit  $Q^2 \rightarrow 0$
- ★  $F_3^{\gamma Z}(x, Q^2)$  should match PDF description at  $Q^2 = 1 \text{ GeV}^2$

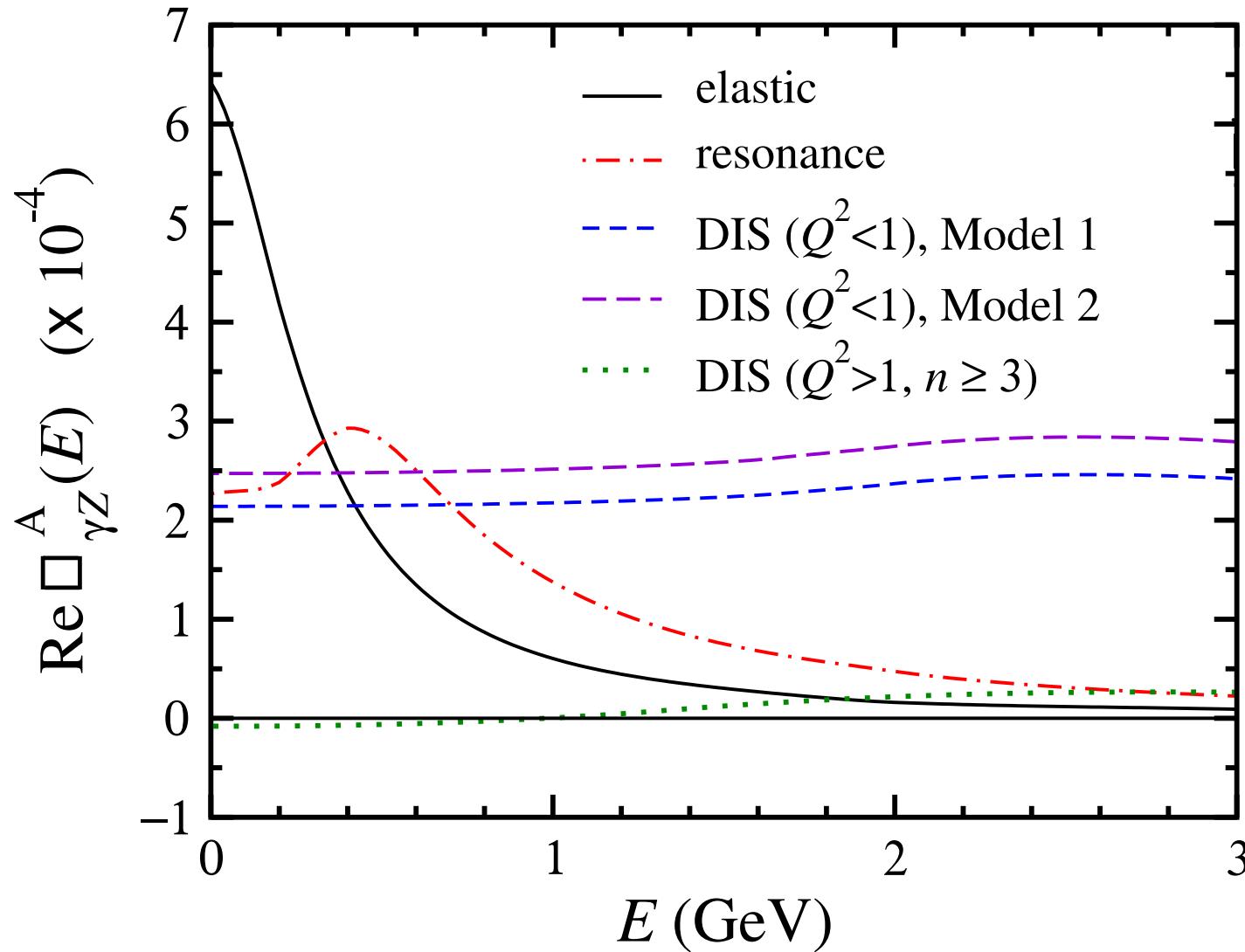
Model 1     $F_3^{\gamma Z}(x, Q^2) = \left( \frac{1 + \Lambda^2/Q_0^2}{1 + \Lambda^2/Q^2} \right) F_3^{\gamma Z}(x, Q_0^2)$

$$F_3^{\gamma Z} \sim (Q^2)^{0.3} \text{ as } Q^2 \rightarrow 0$$

Model 2     $F_3^{\gamma Z}$  frozen at  $Q^2 = 1$  value for all  $W^2$

$$F_3^{\gamma Z} \text{ finite as } Q^2 \rightarrow 0$$

# Axial $h$ correction



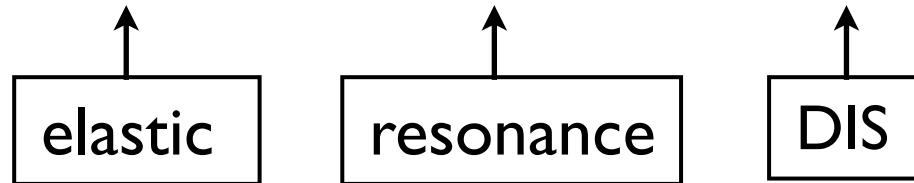
Blunden, WM, Thomas  
PRL 107, 081801 (2011)

→ dominated by  $n = 1$  DIS moment:  $32.8 \times 10^{-4}$   
(weak  $E$  dependence)

# Axial $h$ correction

## ■ correction at $E = 0$

$$\Re \square_{\gamma Z}^A = 0.00064 + 0.00023 + 0.00350 \rightarrow \underline{0.0044(4)}$$



## ■ correction at $E = 1.165 \text{ GeV}$ (Qweak)

$$\Re \square_{\gamma Z}^A = 0.00005 + 0.00011 + 0.00352 = \underline{0.0037(4)}$$

cf.  $\text{MS}^*$  value: 0.0052(5) ( $\sim 1\%$  shift in  $Q_W^p$ )

\* *Marciano, Sirlin, PRD 29, 75 (1984)*

## ■ shifts $Q_W^p$ from 0.0713(8) → 0.0705(8)

## APV in $^{133}\text{Cs}$

- Parity violating dipole transition  $6\text{S}_{1/2} - 7\text{S}_{1/2}$  sensitive to weak mixing angle ( $E \sim 0$ )
  - weak charge of Cs

$$Q_W(\text{Cs}) = 55 \tilde{Q}_W^p + 78 \tilde{Q}_W^n$$

weak charge of *bound p* in Cs nucleus

- Nuclear effect on elastic  $N$  contribution – Pauli blocking
  - intermediate state  $N$  (in target rest frame) must have momentum above Fermi level

$$|\mathbf{q}| > p_F \approx 260 \text{ MeV}$$

$$\Rightarrow Q^2 > Q_{\min}^2 = 2M^2 \left( \sqrt{1 + p_F^2/M^2} - 1 \right) \approx p_F^2$$

# APV in $^{133}\text{Cs}$

## ■ Significantly reduced elastic contribution

$$\square_{\gamma Z}^{p \text{ (el)}} : 0.00064 \rightarrow 0.00029, \quad \square_{\gamma Z}^{n \text{ (el)}} : 0.00044 \rightarrow 0.00020$$

## ■ Total $\gamma Z$ corrections dominated by DIS contributions

	$p$	$n$
total	0.0040(4)	0.0032(4)
MS	0.0052(5)	0.0040(4)
$\Delta \tilde{Q}_W^N$	-0.0012	-0.0008
$\Delta Q_W(\text{Cs})$	-0.065	-0.060

→ overall shift in weak charge (relative to MS)

$$\Delta Q_W(\text{Cs}) = -0.126$$

or 
$$-0.16\% \text{ of } Q_W^{\text{exp}}(\text{Cs}) = -73.20(35)$$

Blunden, WM, Thomas  
arXiv:1208.4310

## Vector $h$ correction

- Vector  $h$  correction  $\square_{\gamma Z}^V$  vanishes at  $E = 0$ , but experiment has  $E \sim 1 \text{ GeV}$  – what is energy dependence?
  - forward dispersion relation

- ★  $\Re e \square_{\gamma Z}^V(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2} \Im m \square_{\gamma Z}^V(E')$
- ★ integration over  $E' < 0$  corresponds to crossed-box, vector  $h$  contribution symmetric under  $E' \leftrightarrow -E'$

- imaginary part given by

$$\begin{aligned}\Im m \square_{\gamma Z}^V(E) &= \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^s dW^2 \int_0^{Q_{\max}^2} \frac{dQ^2}{1 + Q^2/M_Z^2} \\ &\times \left( F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s(Q_{\max}^2 - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)\end{aligned}$$

Gorchtein, Horowitz, PRL 102, 091806 (2009)

# Vector $h$ correction

## ■ $F_{1,2}^{\gamma Z}$ structure functions

### ★ parton model for DIS region

$$F_2^{\gamma Z} = 2x \sum_q e_q g_V^q (q + \bar{q}) = 2x F_1^{\gamma Z}$$

★ in resonance region use phenomenological input for  $F_2$  (e.g. Christy-Bosted), empirical (SLAC) fit for  $R$

- for transitions to  $I=3/2$  states (e.g.  $\Delta$ ), CVC and isospin symmetry give  $F_i^{\gamma Z} = (1 + Q_W^p) F_i^\gamma$
- for transitions to  $I=1/2$  states,  $\gamma\gamma \rightarrow \gamma Z$  rotations fixed by CVC and  $p, n$  helicity amplitudes

$$\frac{\sigma_p^{\gamma Z}}{\sigma_p^{\gamma\gamma}} = (1 - 4 \sin^2 \theta_W) - y_R , \quad y_R = \frac{A_{R, \frac{1}{2}}^p A_{R, \frac{1}{2}}^{n*} + A_{R, \frac{3}{2}}^p A_{R, \frac{3}{2}}^{n*}}{|A_{R, \frac{1}{2}}^p|^2 + |A_{R, \frac{3}{2}}^p|^2}$$

Gorchtein, Horowitz, Ramsey-Musolf [GHRM], PRC 84, 015502 (2011)

# Vector $h$ correction

## ■ $F_{1,2}^{\gamma Z}$ structure functions

- ★ for background at low  $Q^2$ , assume vector meson dominance (following Gorchtein *et al.*)

$$\frac{\sigma^{\gamma Z}}{\sigma^{\gamma\gamma}} = \frac{(2 - 4 \sin^2 \theta_W) - 4 \sin^2 \theta_W R_\omega + (3 - 4 \sin^2 \theta_W) R_\phi + \zeta_R R_C}{1 + R_\omega + R_\phi + R_C}$$

↑  
“continuum”

$$R_V = \frac{\sigma^{\gamma^* p \rightarrow V p}}{\sigma^{\gamma^* p \rightarrow \rho p}}$$

ratio of production cross sections  
for vector meson  $V$  to  $\rho$  meson

- parameter  $\zeta_R$  not constrained in VMD
- GHRM assume  $\zeta_R = 1 \pm 1$

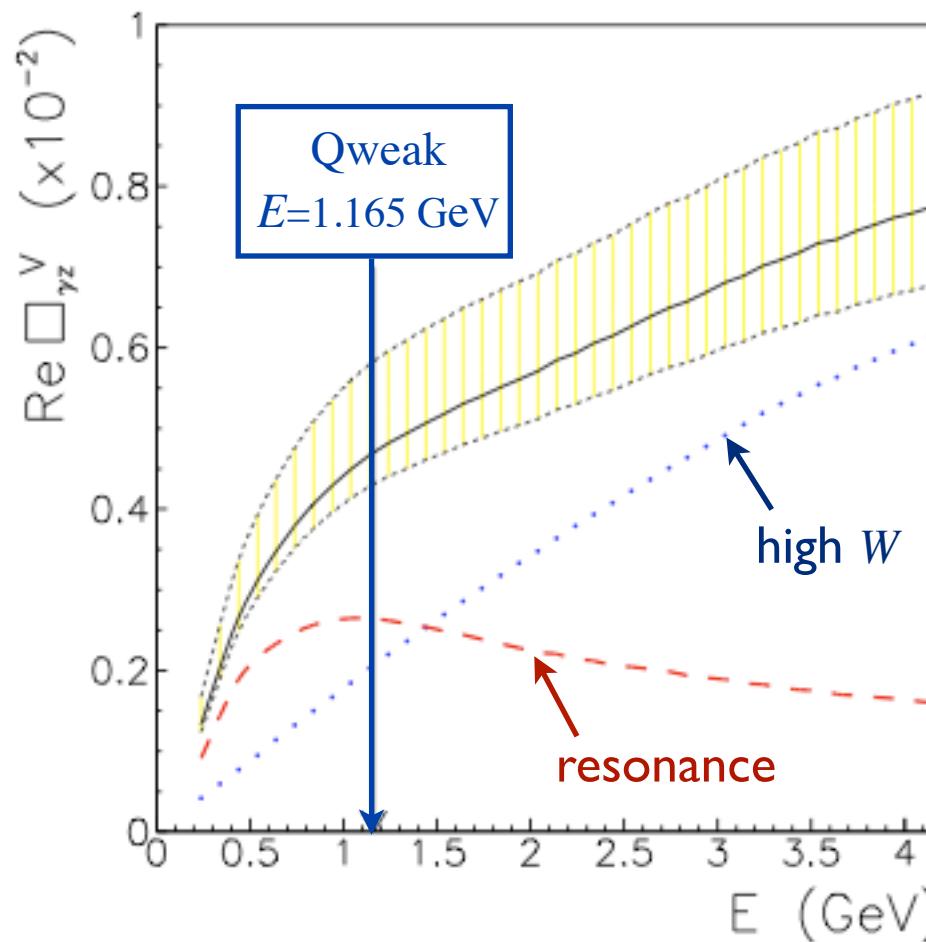
largest source of error!

# Previous calculations

## ■ Total $\square_{\gamma Z}^V$ correction

$$\Re \square_{\gamma Z}^V = 0.0047^{+0.0011}_{-0.0004}$$

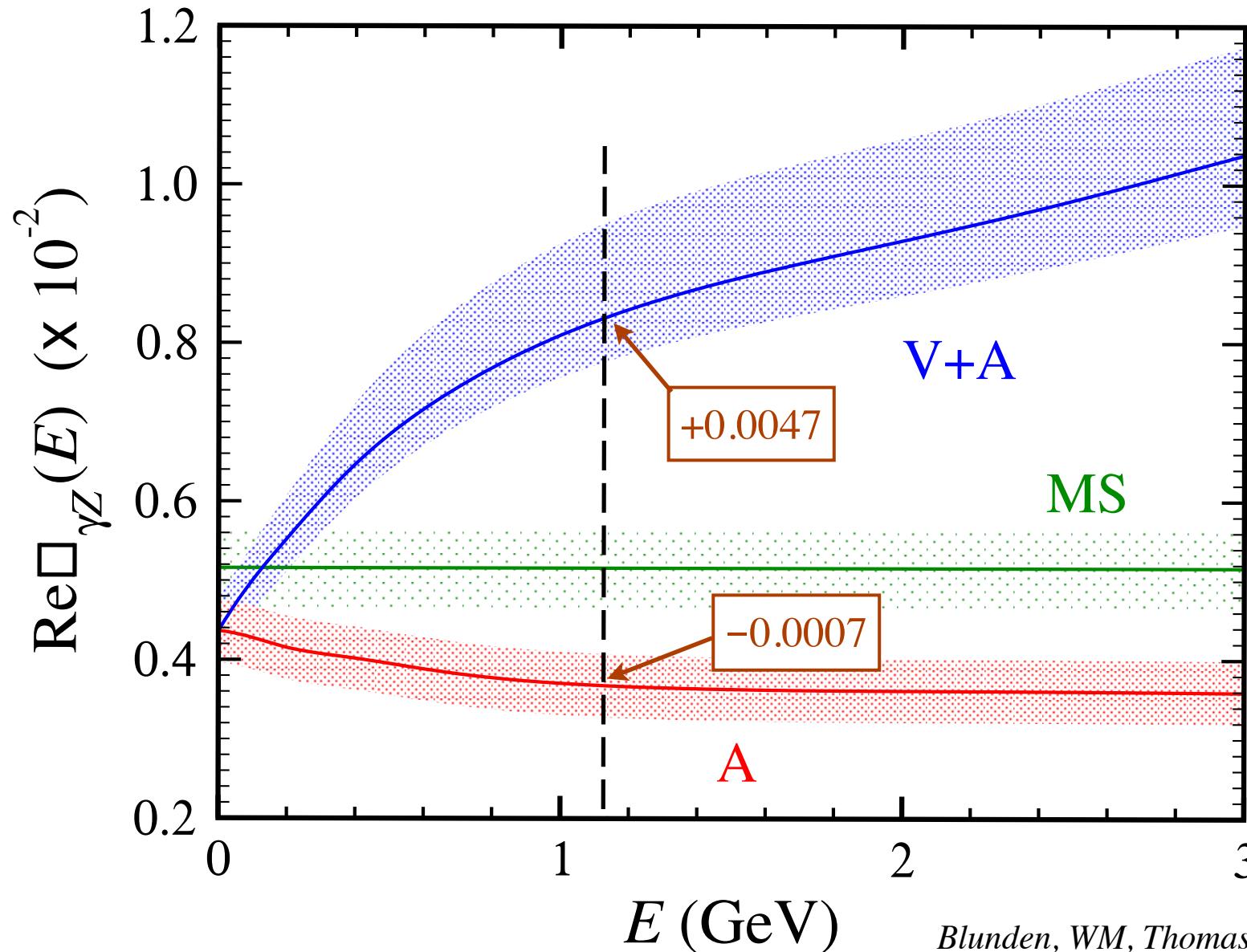
or  $6.6^{+1.5}_{-0.6}\%$  of uncorrected  $Q_W^p$



Sibirtsev, Blunden, WM, Thomas  
PRD 82, 013011 (2010)

# Combined vector and axial $h$ correction

$$Q_W^p = 0.0713 \rightarrow 0.0705 \quad (\text{at } E=0)$$



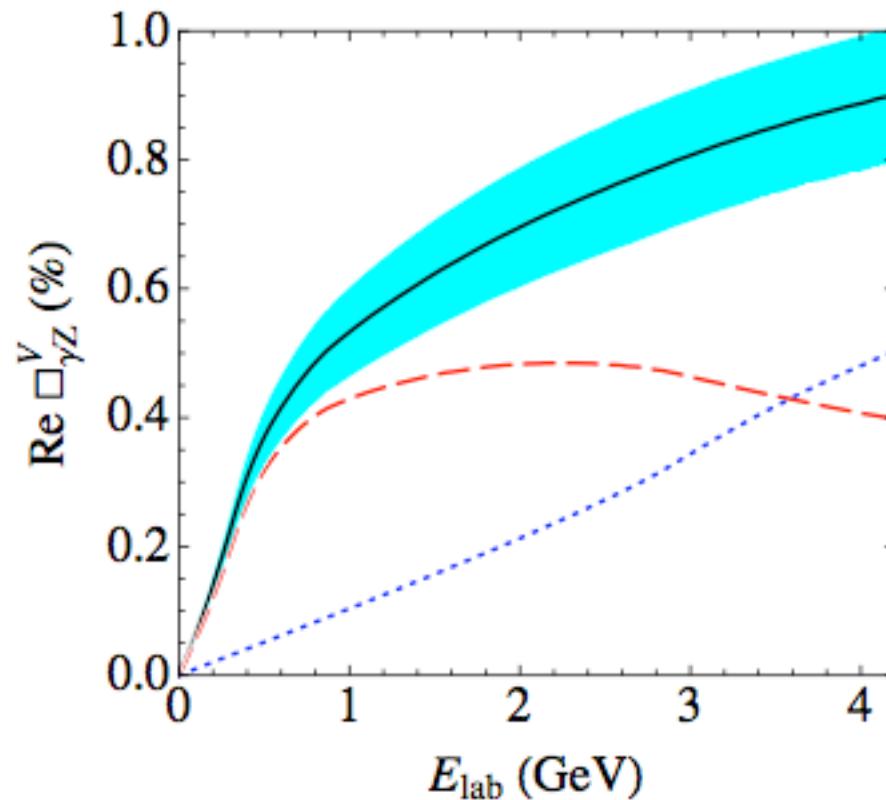
At  $E=1.165$  GeV,  
 $E$ -dependent  
correction is  
 $+ \underline{0.0040}$

Blunden, WM, Thomas, PRL 107, 081801 (2011)

# Previous calculations

## Rislow & Carlson

$$\Re e \square_{\gamma Z}^V = 0.0057 \pm 0.0009$$



Rislow, Carlson  
PRD 83, 113007 (2011)

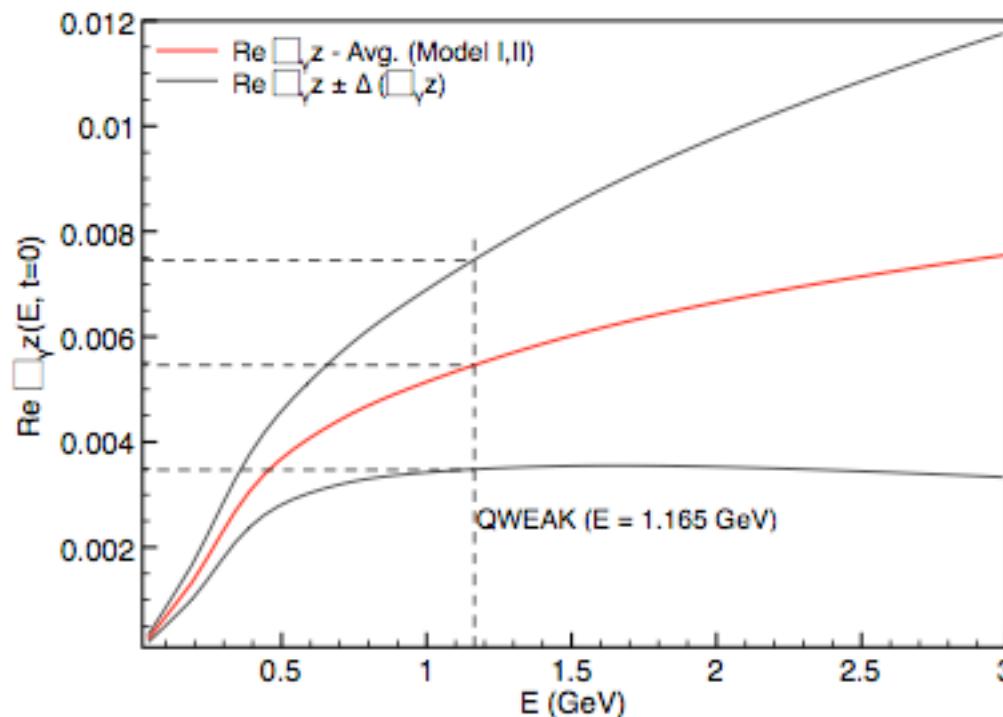
→ compatible with SBMT within errors

# Previous calculations

## Gorchtein, Horowitz & Ramsey-Musolf

$$\Re e \square_{\gamma Z}^V = (5.39 \pm 0.27 \pm 1.88 \begin{array}{l} +0.58 \\ -0.49 \end{array} \pm 0.07) \times 10^{-3}$$

↑  
model      bckgnd      res.      ↑  
                        ↑  
                        ↑  
*t* dep.



*Gorchtein, Horowitz,  
Ramsey-Musolf  
PRC 84, 015502 (2011)*

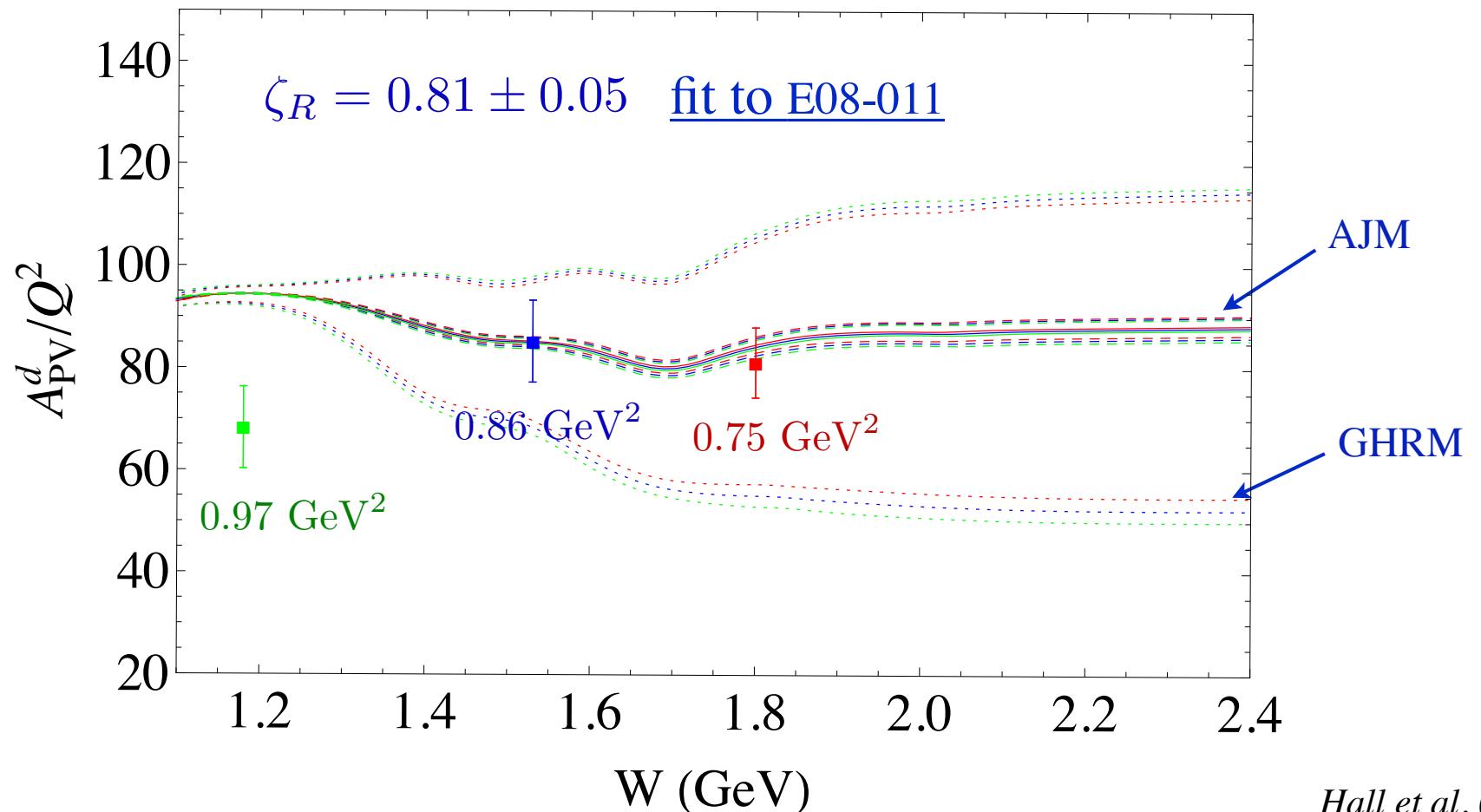
→ central value consistent with SBMT and RC,  
but 2 x larger uncertainty

# Can the $\gamma Z$ interference be constrained by other observables?

- Parity violating inclusive DIS asymmetries
- Parton distribution functions from global fits

## ■ Constraints from PVDIS asymmetries (E08-011 on deuterium\*)

$$A_{\text{PV}} = g_A^e \left( \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e} (y - y^2/2)xF_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$

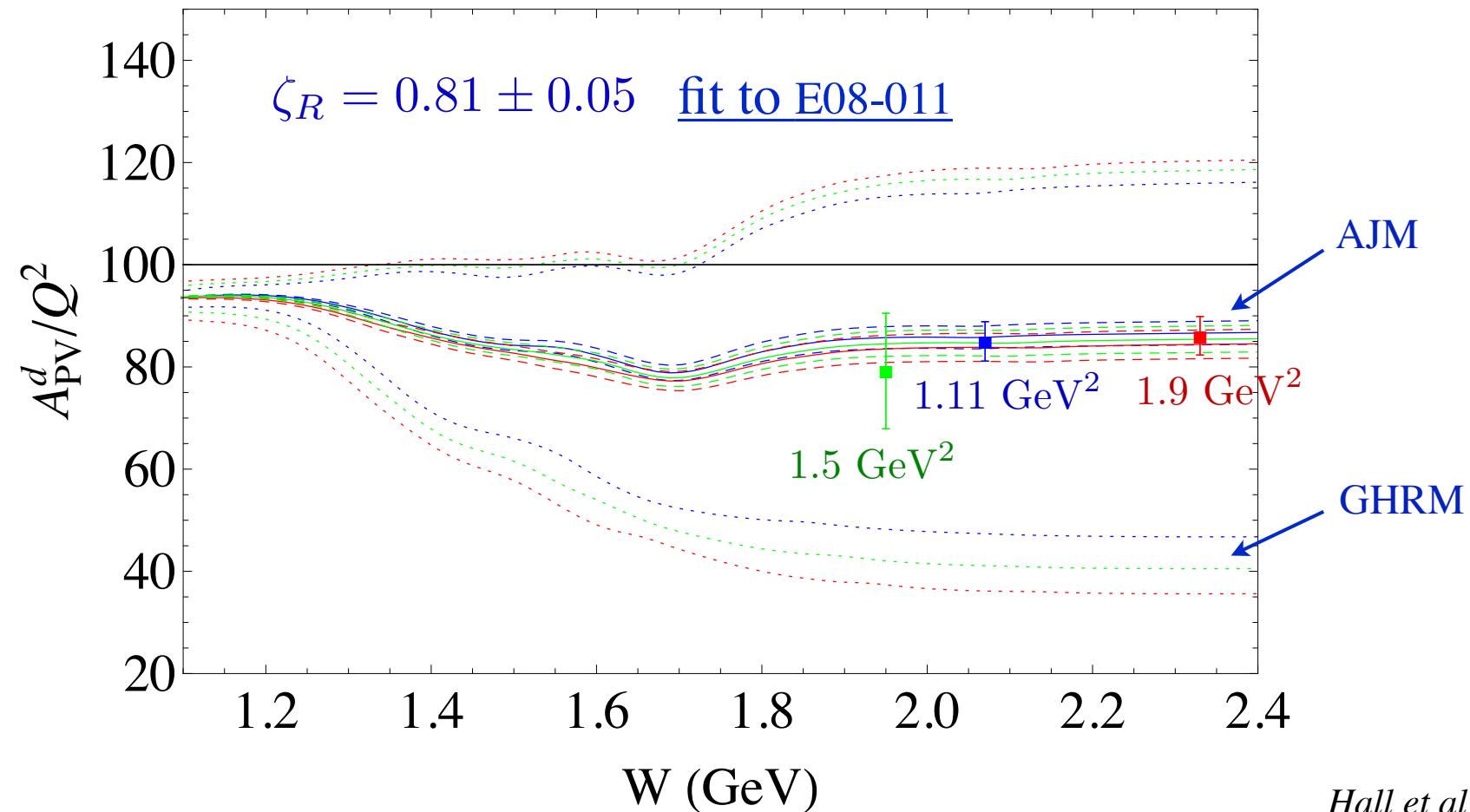


Hall et al. (2012)

\* Preliminary, X. Zheng et al. (2012)

## ■ Constraints from PVDIS asymmetries (E08-011 on deuterium)

$$A_{\text{PV}} = g_A^e \left( \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e} (y - y^2/2)xF_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$

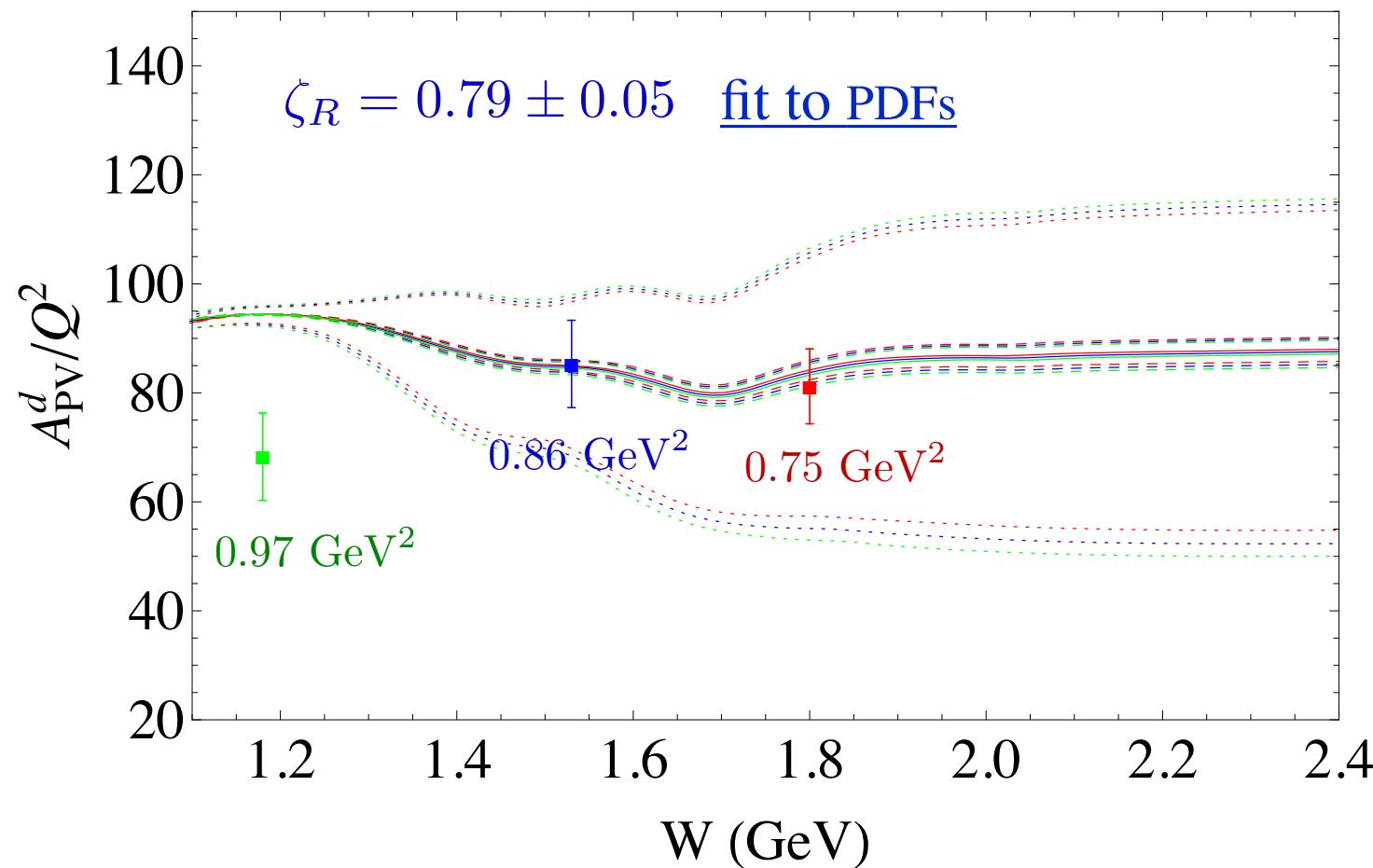


Hall *et al.* (2012)

\* Preliminary, X. Zheng *et al.* (2012)

## ■ Constraints from PVDIS asymmetries (E08-011 on deuterium)

$$A_{\text{PV}} = g_A^e \left( \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e} (y - y^2/2)xF_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$

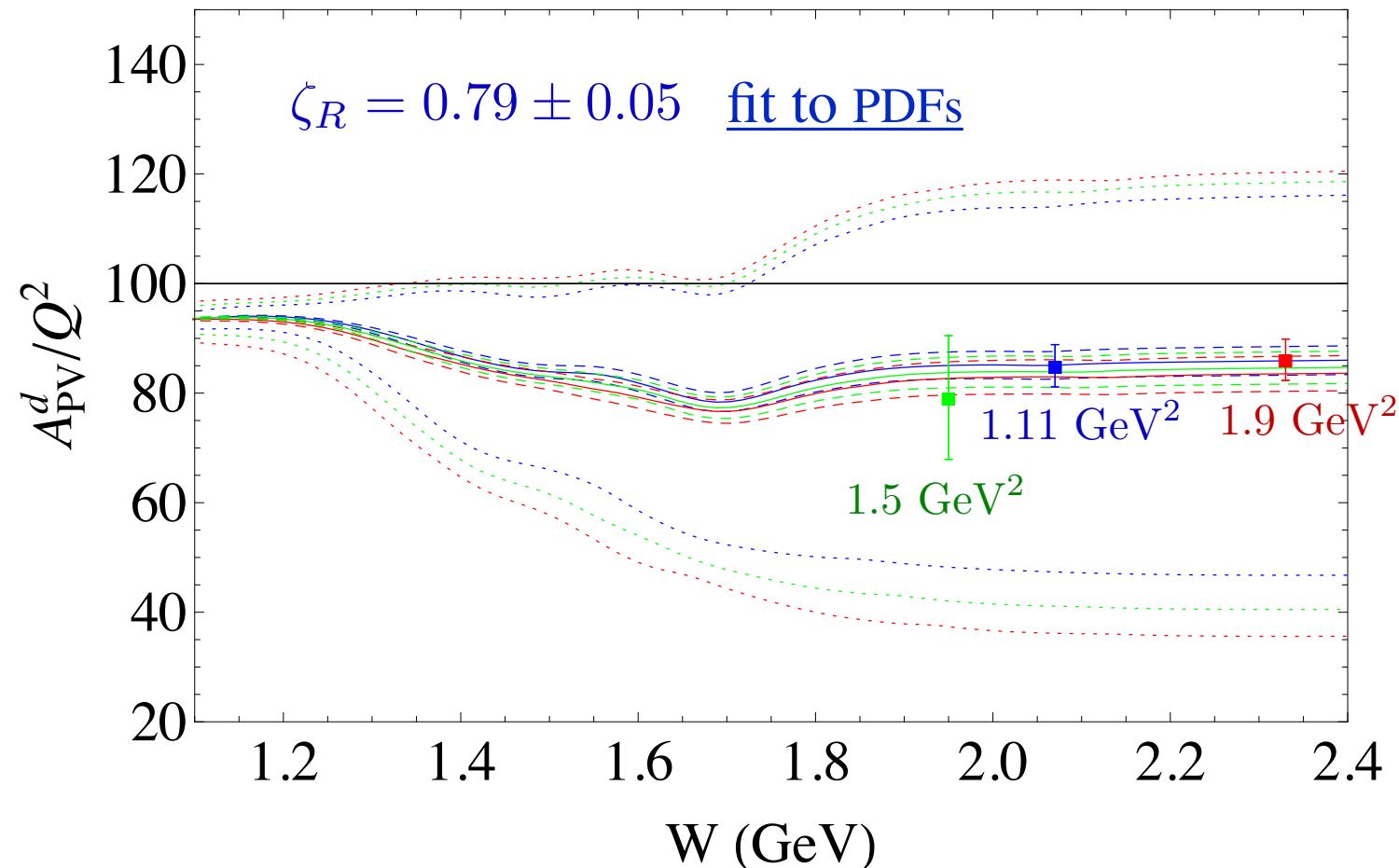


Hall *et al.* (2012)

\* Preliminary, X. Zheng *et al.* (2012)

## ■ Constraints from PVDIS asymmetries (E08-011 on deuterium)

$$A_{\text{PV}} = g_A^e \left( \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e} (y - y^2/2)xF_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$

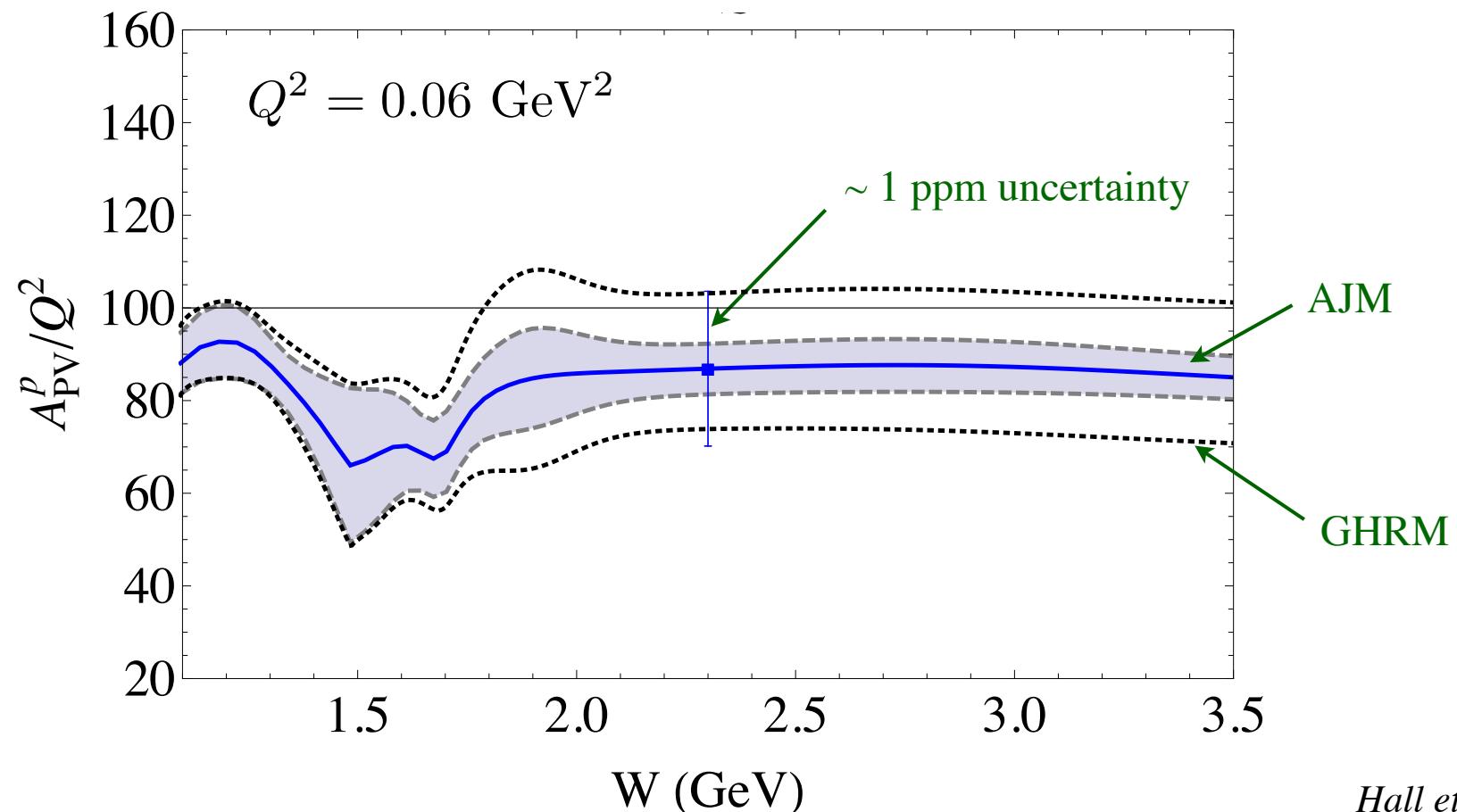


Hall *et al.* (2012)

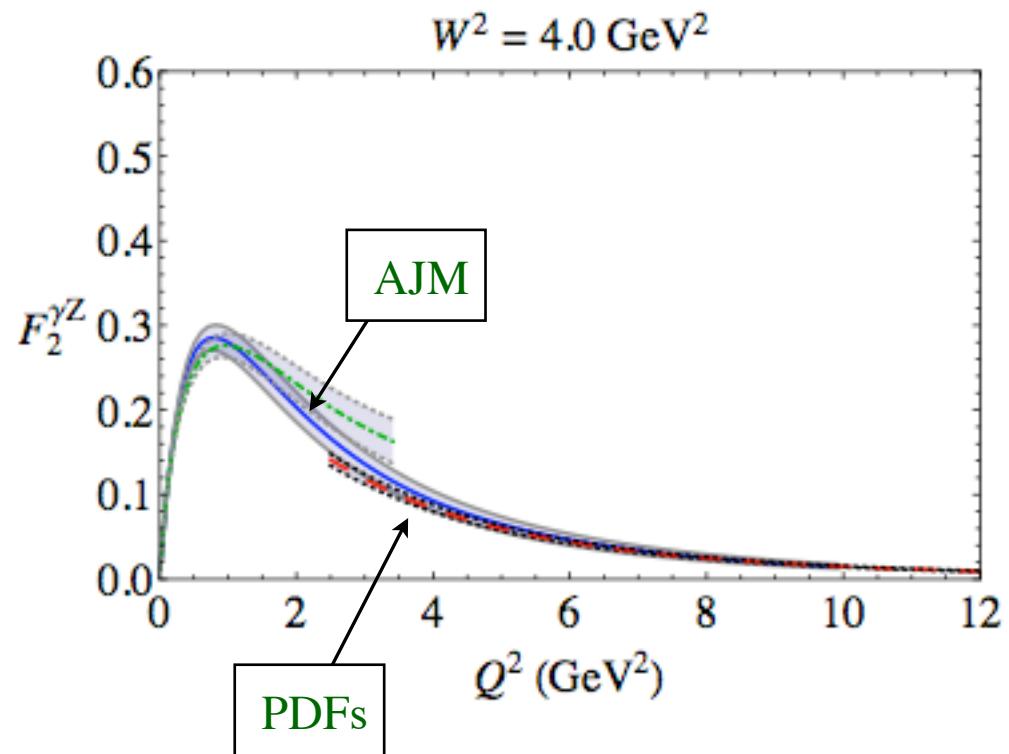
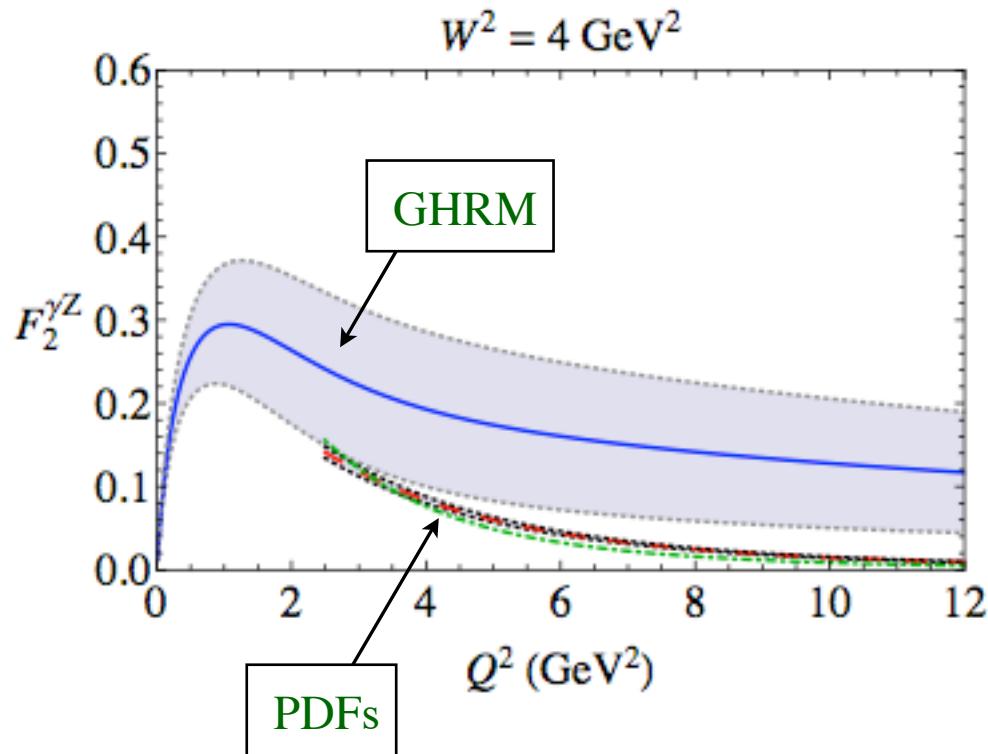
\* Preliminary, X. Zheng *et al.* (2012)

## ■ Expected inelastic asymmetry data from Qweak

$$A_{\text{PV}} = g_A^e \left( \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \right) \frac{xy^2 F_1^{\gamma Z} + (1-y) F_2^{\gamma Z} + \frac{g_V^e}{g_A^e} (y - y^2/2) x F_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y) F_2^{\gamma\gamma}}$$

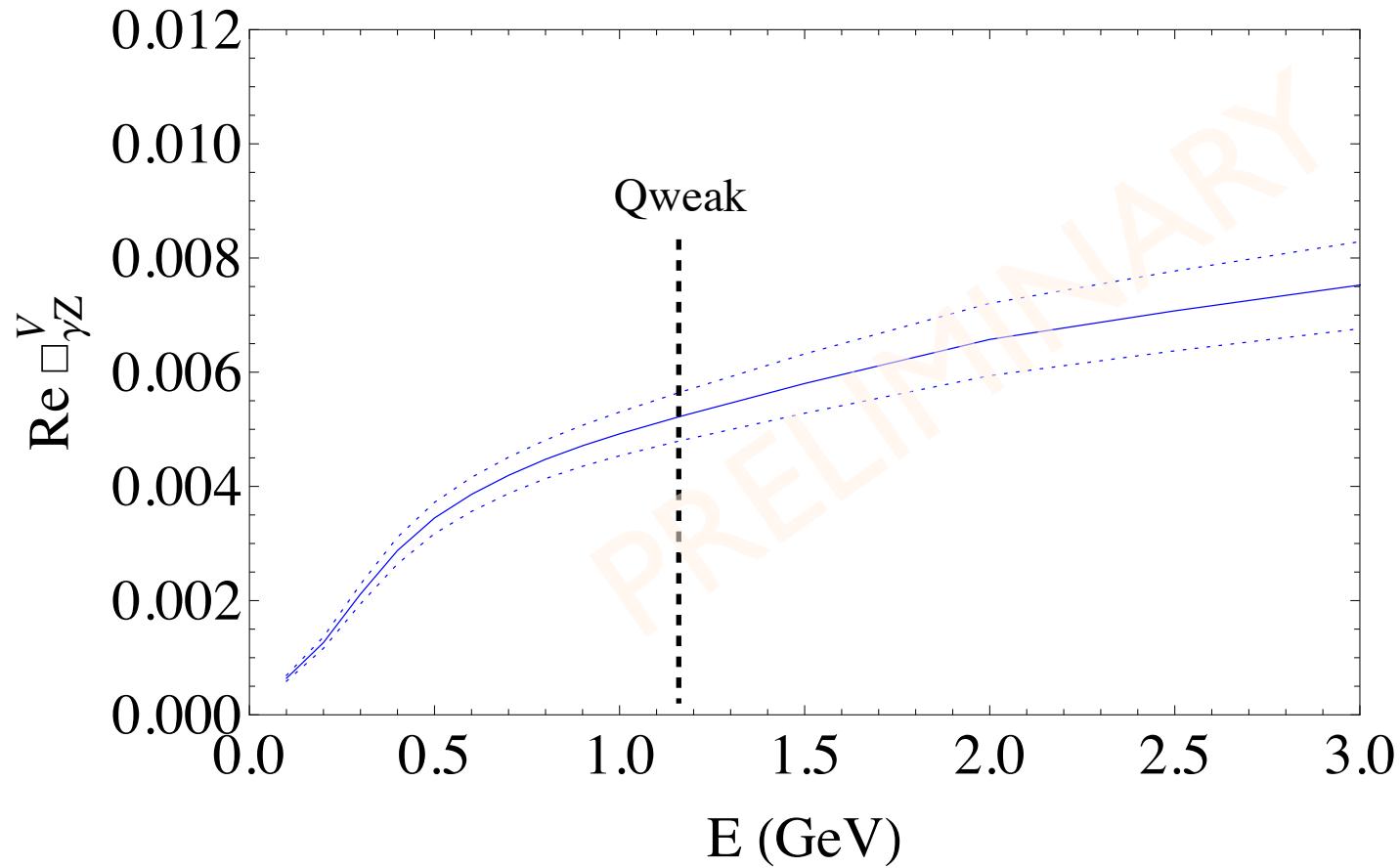


## ■ Constraints from global PDFs



→ significant constraints on high-mass “continuum” part of non-resonant structure function from global PDF fits

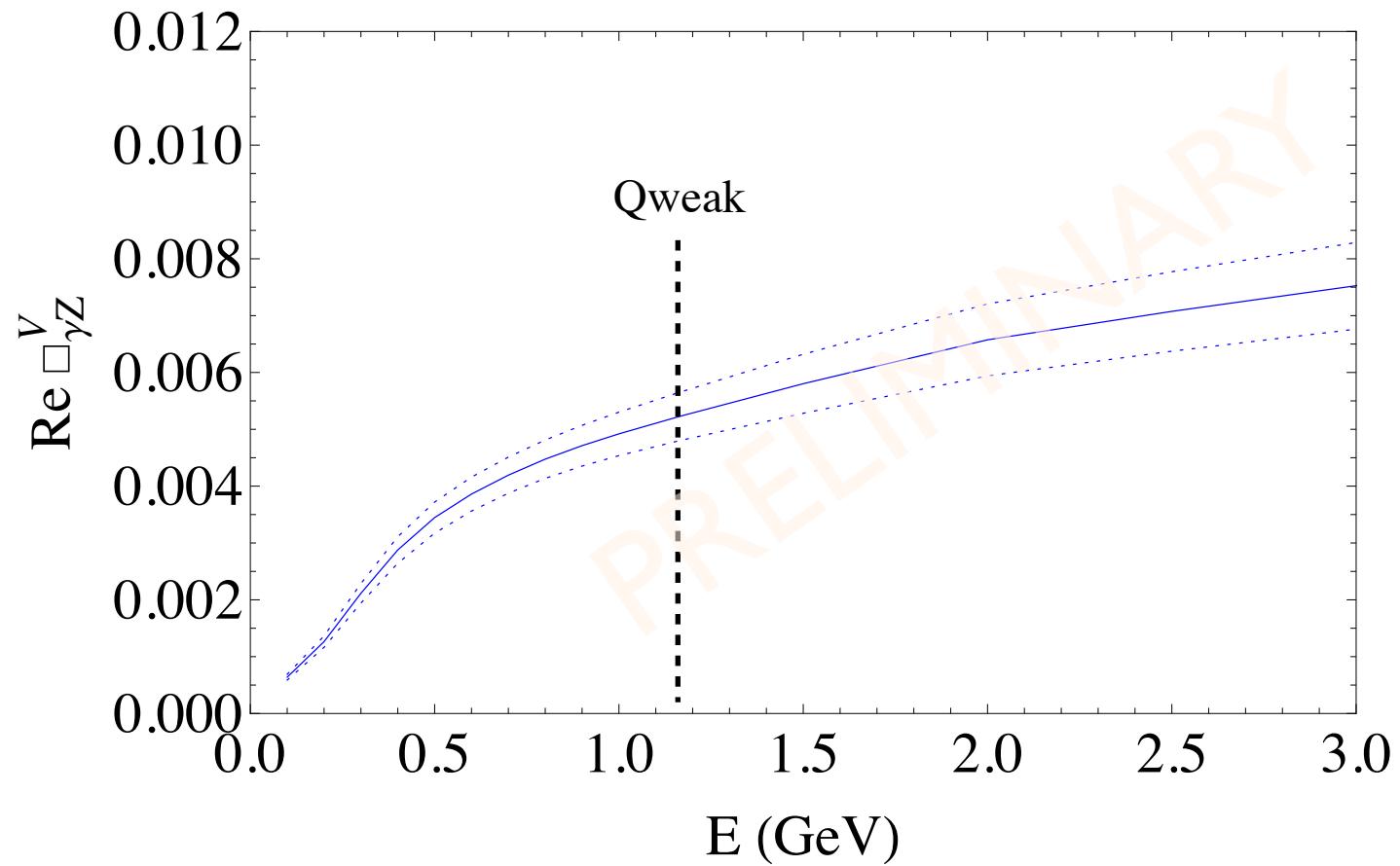
## ■ Total $\square_{\gamma Z}^V$ correction



$$\Re \text{e } \square_{\gamma Z}^V = (5.45^{+0.15}_{-0.17} \pm 0.27 \pm 0.02) \times 10^{-3}$$

$\uparrow$  bckgnd       $\uparrow$  res.       $\uparrow$  DIS

## ■ Total $\square_{\gamma Z}^V$ correction

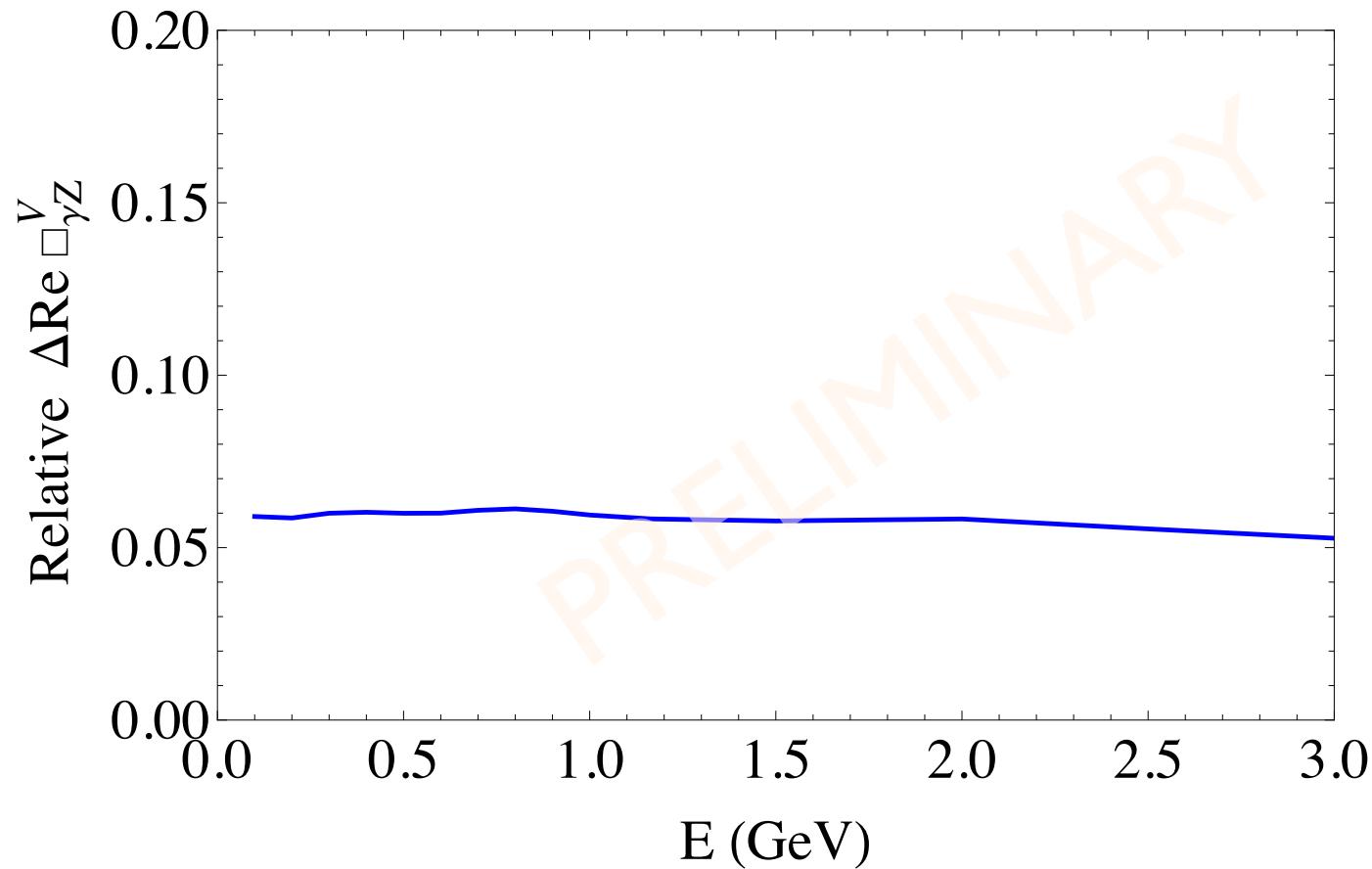


Hall et al. (2012)

$$\Re \text{e } \square_{\gamma Z}^V = (5.45^{+0.31}_{-0.32}) \times 10^{-3}$$

→ uncertainty  $\sim 6$  times smaller cf. GHRM

## ■ Relative uncertainty



→ very mild energy dependence

# Summary

- $\gamma Z$  box corrections computed via dispersion relations from inclusive  $\gamma Z$  interference structure functions
  - new formulation in terms of moments puts on firm footing earlier estimates within “free quark model”
- Axial-vector hadron  $\gamma Z$  corrections to APV in  $^{133}\text{Cs}$ 
  - shift relative to MS value for  $Q_W(\text{Cs})$  of  $-0.16\%$   
 $(\Delta \sin^2 \theta_W \approx 4 \times \text{SM uncertainty})$
- Significant constraints on vector hadron correction from new PVDIS asymmetry data & global PDF fits
  - reduces uncertainty on  $\Re e \square_{\gamma Z}^V$  by factor  $\sim 6$  cf. GHRM
  - additional PVDIS data would further constrain  $\Re e \square_{\gamma Z}^V$